Recent surrogate modeling approaches in electromagnetic nondestructive evaluation

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***Abstract*—Surrogate modeling is gaining more and more ground in various engineering domains. The use of computationally cheap surrogate models in electromagnetic nondestructive evaluation –where usually heavy numerical simulators are involved– has already shown and still promises considerable improvement over traditional approaches for specific applications. The purpose of this paper is to propose an (limited) overview of such an approach in the framework of electromagnetic nondestructive evaluation. Adaptive sampling methods for the solution of inverse problems and for the generation of "optimal", interpolation-based surrogate models are presented and illustrated by examples drawn from eddy-current nondestructive testing.**

***Index Terms*—Surrogate modeling; Nondestructive evaluation; Eddy-current testing; Kriging**

I. Introduction

Electromagnetic nondestructive evaluation (ENDE) is widely used in industry to reveal and characterize in-material defects. The solution of the inverse problem (i.e., determining the properties of the defect based on the knowledge of some measured data) is of main interest, however, it is still a challenging issue. Besides the theoretical pitfall of possibly being ill-posed, inverse problems raise computational difficulties as well, since inversion is often performed via solutions of direct problems. Even if sophisticated numerical simulators providing fine precision are available in ENDE (e.g., finite element or integral equation schemes) their related computational burden is high.

In this paper, kriging-assisted surrogate modeling approaches are presented. Kriging is a tool for function approximation (see, e.g., the textbook [1]): based on some observations, an unknown function can be predicted at unobserved locations within a stochastic framework (via a Gaussian random process model). Besides the prediction, its estimated uncertainty is also provided. In electromagnetics, kriging has already shown good performance (e.g., [2], [3], [4] as examples for single objective optimization or [5] as a recent approach for multiobjective optimization).

II. The studied simple ECT setup

The approaches are illustrated by a 2-parameter eddy-current testing (ECT) example. A homogeneous, non-magnetic conductive plate is affected by a thin, rectangular-shaped crack. An air-cored pancake type coil driven by time-harmonic current scans above the plate (Fig. 1). The variation of the coil impedance $\Delta\mathcal{Z}(t)$ is measured (t is the coil position over a rectangular surface). The position and the orientation of the crack are known, only its length L and depth D are enabled to vary. For the numerical simulation, an integral formalism is used [6]. The varying crack properties, denoted by $\mathbf{x} = [L, D]$, are called the *input* of the simulation and the corresponding simulated impedance variation is denoted by $\Delta Z_{\mathbf{x}}(t)$.

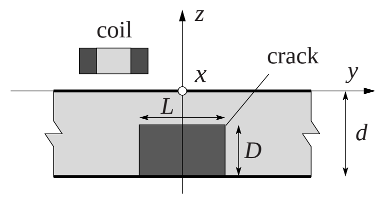

Figure 1: Cross-section of the studied ECT configuration. The depth D is given in percentages of the plate thickness d.

III. Optimization-based inversion by the EGO algorithm

We present a way for the solution of the inverse problem, via the traditional optimization task:

$$\mathbf{x}^\star = \arg\min Q(\mathbf{x}), \text{ where } Q(\mathbf{x}) = \frac{||\Delta\mathcal{Z}(t) - \Delta Z_{\mathbf{x}}(t)||}{||\Delta\mathcal{Z}(t)||}. \quad (1)$$

Since the objective function $Q(\mathbf{x})$ is "expensive-to-evaluate" (needing a numerical EM simulation) and might have many local minima, the so-called "Efficient Global Optimization" (EGO) algorithm [7] has been applied. Also the authors of the present paper have dealt with the EGO algorithm in the context of ECT inverse problems [8].

The main idea of EGO is to construct the cheap surrogate model of the objective function by kriging. Based on this prediction, along with its estimated uncertainty, a sequential sampling method is built up: $Q(\mathbf{x})$ is being evaluated step-by-step always at the most "promising" point $\mathbf{x}$ in an iterative loop. The way to choose the next point is a compromise between a local and a global search over the input domain. The performance of EGO in our example is shown (Fig. 2). After 10 initial observations (by a Latin Hypercube Sampling) and 10 more iterations the global minimizer of $Q(\mathbf{x})$ is found.

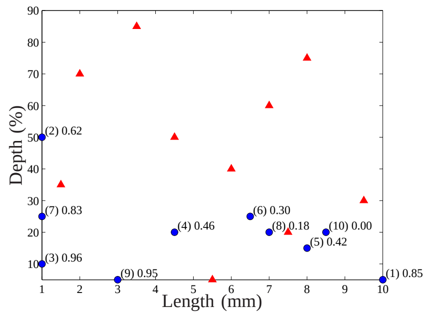

Figure 2: Performance of the EGO algorithm: samples are shown over the input domain. Triangles: initial samples; dots: adaptively added samples, number of iteration is in brackets followed by the actual value of Q. (The "measured data" – related to a $L = 8.5\,\mathrm{mm}$, $D = 20\,\%$ crack– is computed by the numerical simulator).

IV. Adaptive databases as surrogate models

Whereas the EGO concentrates the observations into certain "promising" regions of the input domain (according to the specific inverse problem to be solved), one can perform a sampling also to achieve a general surrogate model to interpolate the output signal $\Delta Z_{\mathbf{x}}(t)$ based on samples stored in a database. However, the precision of the yielded surrogate model depends not only on the interpolator but also on the choice of the samples. Such adaptive sampling strategies, aiming to reduce the interpolation error $\varepsilon(\mathbf{x}) = \left\| \Delta Z_{\mathbf{x}}(t) - \widehat{\Delta Z}_{\mathbf{x}}(t) \right\|$ (where $\widehat{\Delta Z}_{\mathbf{x}}(t)$ is given by the surrogate model) are then proposed.

When the nearest neighbor rule is used as an interpolator, the sampling must be uniform in the domain of impedance signals. A kriging-based sampling strategy has been developed to generate such adaptive databases. It is presented in detail in [9], along with further applications (e.g., the structure of these databases provides meta-information on the studied problem). Only the performance of the approach is illustrated herein via the 2-parameter ECT example (Fig. 3).

More precise surrogate models can be constructed by using more sophisticated interpolators. The authors presented the use of a *functional kriging* interpolator [10]. In so doing, even a naive sampling provides fine approximations, however, sample-sets can also be generated adaptively based on the estimated uncertainty of the functional kriging prediction.

V. Conclusion

In some applications surrogate models seem to be promising alternatives for time-consuming numerical simulations in nondestructive evaluation. All above mentioned approaches are outlined in more detail and illustrated by more complex ECT examples (involving up to 6 parameters) in the full version of the paper.

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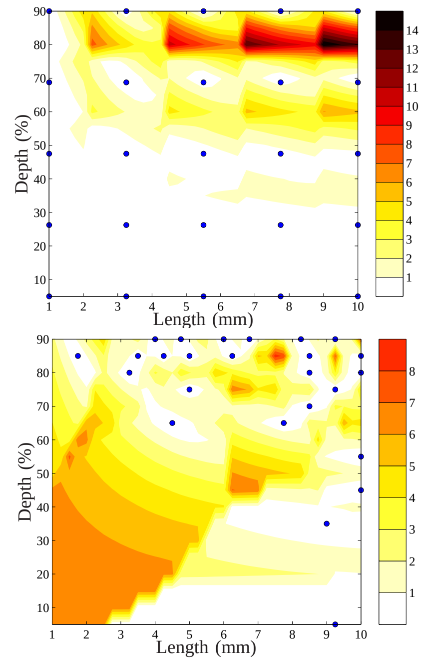

Figure 3: Interpolation error $\varepsilon(\mathbf{x})$ normalized by the norm of the signal of a $L = 8.5\,\mathrm{mm}$, $D = 20\,\%$ crack (colormap). Top: naive sample distribution. Bottom: result of our adaptive sampling (more samples are concentrated in the regions where the problem is more sensitive to the input parameters, this is why the error is smaller in this case).

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